Sensitivity Analysis of Melting and Freezing of Snow on Roofs

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Abstract. The paper describes a statistical analysis of a mathematical model for calculation of the melting and freezing of snow on roofs. Parameters are roof length, overhang length, heat resistance of roof and overhang, outdoor and indoor temperature, snow thickness and thermal conductivity. If the snow thickness is above a limit value, then part of the snow will melt. This gives water flow to the overhang. Part of the water will freeze on the overhang and a part will drip from the roof. If the water flow is small, then all the water will freeze on the overhang otherwise there will be dripping and icicles.

The paper uses sensitivity analysis with the Morris method to find parameters that are negligible, linear or non-linear. The Sobol sensitivity indices are also calculated. By means of sensitivity analysis, it is possible to determine, which parameters are the most important e.g. the thickness of the snow or indoor and outdoor temperatures and roof thermal resistance. In practice, some parameters are difficult to change, but the analysis shows where the effect is most efficient.

Keywords: Snow, Statistical Analysis, Roofs.

1 Introduction

In winter snow will accumulate on the roof of buildings. This will add insulation to the roof if the snow does not melt. In most periods part of the snow will melt and the water flow can generate icicles at the eaves. This occurs if there is solar radiation on the roof, as it will melt part of the snow. But this is not the only important factor. Investigations in United States by Tobiasson [1] showed that the most important factor for the generation of icicles is melting caused by a heated building. The problem is smaller for a thermally well-insulated building. An unheated building presents few problems. A Canadian report by Straube [2] discusses formation of ice dams on the roof as the melting water can freeze on the roof overhang. Melting snow on glass roofs has been discussed in Nielsen [3] and [4]. It is therefore interesting to analyze melting of the snow on the roof, freezing on the overhang and dripping from the eave.

For analyzing the problem with snow melting and freezing the most important factors are climate and the insulation level of the roof. A HVAC system in the building can worsen the problems. An example from Norway [5] is a low building where large icicle were formed. In this case the ventilation system was placed in the attic above the room. The thermal insulation was on the attic floor. As there where little or no thermal insulation of the ventilation system the heat from the air ducts and heat exchangers gave off so much heat that it increased the attic temperature. The result was that the snow on the roof melted in cold periods and large icicles were created all the way from the gutter to the ground. The solution for the problem was thermal insulation of the technical systems.

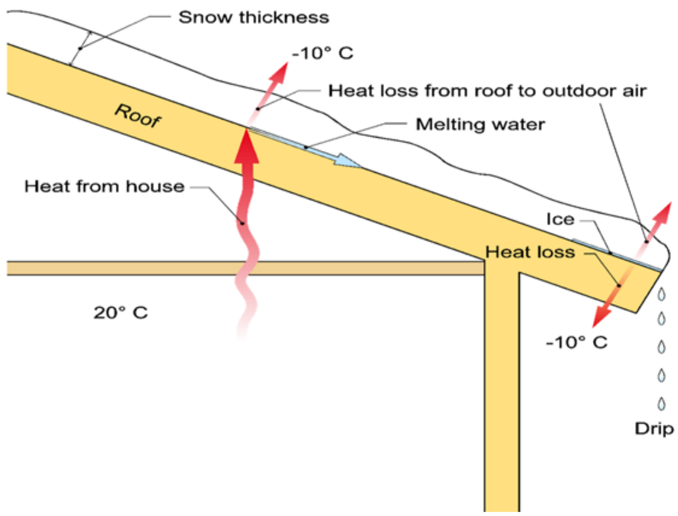

Fig. 1. Heat balance and water flow on roof

2 Energy Balance – Calculation Method

Figure 1 is a sketch of the energy and water balance of a typical roof with snow in the winter. We assume that the attic is not ventilated. The effect of a ventilated attic is discussed later. The heat flows from the interior of the house to the surface of the roof through the roof construction. The heat will flow through the snow to the outdoor air and possibly melt some of the snow. With small snow thicknesses, there will be no melting of the snow as the surface temperature on the roof is below 0 ºC. With large snow thicknesses the bottom of the snow layer will melt and result in melt water that follows the slope of the roof. It is therefore interesting to calculate the snow thickness that corresponds to the limit between melting and no melting. This is the case if the roof surface temperature is 0 ºC. This is called the melting limit. With known indoor and outdoor temperatures as well as the U-value of the roof, the snow melting thickness can be found. Part of the melting water will freeze on the overhang and the rest will drip from the roof. A special case is if all the melting water is freezing on the overhang so there is no dripping. This is called the dripping limit. It is possible to calculate the snow thickness for this case. This thickness is always larger than the melting limit.

The calculation of the energy balance and the water flows is performed as follows; The heat flows Q is pr. unit of width of the roof. The thermal conductivity of the snow is calculated from the snow density. The thermal resistance of the snow layer (R_s) is then:

$$R_s = \text{snow thickness} / \text{snow thermal conductivity} \quad (1)$$

The heat flow (Q_r) from the building to the upper surface of the roof and the melting snow (with a temperature of 0 °C) is:

$$Q_r = l_r / R_r * (T_i - 0) \quad (2)$$

The length of the roof is (l_r). The indoor temperature is (T_i). The thermal resistance between the room and upper surface of the roof is (R_r). The heat flow (Q_s) from the underside of the snow to the outside air (T_u) is:

$$Q_s = l_r / R_s * (0 - T_u) \quad (3)$$

The thermal resistance of the snow is (R_s). The heat flow (Q_m) for melting is:

$$Q_m = Q_r - Q_s \quad (4)$$

The value (Q_m) must be positive or zero (no melting). An interesting temperature T_{melt}) is the theoretical surface temperate of the roof if we ignore the melting:

$$T_{melt} = T_i - (R_r / (R_r + R_s)) * (T_i - T_u) \quad (5)$$

If $T_{melt} > 0$ then use formula (6) to calculate the flow of melting water m_{melt} else m_{melt}=0:

$$m_{melt} = l_r / MH * (T_i / R_r + T_u / R_s) \quad (6)$$

MH is the latent melting heat from ice to water. The amount of water freezing m_{freeze} on the overhang can be calculated from formula 7:

$$m_{freeze} = -l_o / MH * (T_u / R_s + T_u / R_o) \quad (7)$$

The length of the overhang is (l_o). The resistance from the surface of the roof to the outside air under the overhang is (R_o). The amount of water dripping m_{drip} is calculated with formula 8.

$$m_{drip} = m_{melt} - m_{freeze} \quad (8)$$

The amount of water dripping can either drip or freeze as icicles. In case we have a gutter, then a part will freeze in the gutter and the rest will go into the drainpipe.

In the model is the outdoor heat resistance set to zero instead of using the normal 0,04 m^2K/W. That is acceptable as the heat resistance of the snow in most cases is 20-100 times larger. An alternative is to make a small change in either the snow thickness or the outdoor temperature to account for the heat resistance between the roof and outdoor. Using a model with convection and radiation heat transfer complicate the model without changing the result of the statistical analysis.

2.1 Melting and dripping limits

The melting limit and dripping limit can be calculated from these equations. In table 1 is an example for 20 °C indoor and -10 °C outdoor temperature and a roof length of 7 m an overhang of 0.4 m.

Table 1. Examples of limits

Parameter	U-value roof	Melting limit	Dripping limit
Old house	1 W/m^2K	3 cm	3.3 cm
Insulated house	0.3 W/m^2K	10 cm	12 cm
New insulated house	0.15 W/m^2K	20 cm	31 cm
Future house	0.1 W/m^2K	30 cm	63 cm

It is seen that for an old house the dripping limit is nearly the same as the snow melting limit. For a new insulated house the dripping limit is much higher than the melting limit, as a larger part of the melt water will freeze on the overhang. The result is that a well-insulated house has a lower risk of icicles and an overhang is important to reduce the risk of icicles. More information is found in [6].

2.2 Non-stationary cases

The calculations shown are based on stationary conditions. In reality, the amount of melting water will be reduced when the snow melts since the thickness is then reduced over time. A mathematical model for dynamic conditions can be found in Claesson and Nielsen [7]. This model has also been expanded to include a case with a roof window in the construction. This will typically give more melting water as the U-value of the window is higher than the roof U-value and will result in higher risk of icicles.

3 Methods for Sensitivity Analysis

Sensitivity analysis can be performed using different methods. In this case we use the Morris and Sobol methods as described in Saltelli [8]. The following gives a short description of the methods. For a more detailed description of the methods see the more specialized statistical literature mentioned in Saltelli..

The statistical simulation is done with the Sensitivity Analysis Library SALib [9]. SALib is an open source library written in Python for performing sensitivity analysis. SALib provides a workflow where the mathematical model (in this case the energy balance of the roof) is in a separate module, which in this case is also written in Python [10]. SALib is responsible for generating the model inputs, using one of the sample functions, and computing the sensitivity indices from the model outputs, using one of the analyze functions. A sensitivity analysis using SALib has four steps:

1. Determine the model inputs (parameters) and their sample range.
2. Run the sample function to generate the model inputs.
3. Evaluate the model using the generated inputs, saving the model outputs.
4. Run the analyze function on the outputs to compute the sensitivity indices.

The second part generates for instance 1000 cases, which are run through the model in the third part and generate a similar number of output results. There are special sample and analyze functions for each method as Morris or Sobol.

3.1 Morris method

The Morris method is a screening method to find parameters that are important or negligible. The method varies one parameter at a time. Each parameter is divided into a discreet number of values that are chosen within the range of variation. The method calculates two sensitivity measures for each parameter. The measure for the overall effect, Morris μ of the parameter on the output, can be called the mean value. The other measure, Morris σ estimates the second and higher order effects in which the parameter is involved and can be called the standard deviation for the parameter.

The method calculates Morris μ and σ for each parameter. A high μ indicates a parameter with an important overall influence on the output. A high σ indicates either interaction with other parameters or a parameter that is non-linear. If both μ and σ are low then this parameter is negligible. The method tends to be qualitative as for ranking the input parameters in order of importance.

The calculation from the statistical module also gives a μ*, that is the absolute mean as a supplement to the normal μ value. The μ* is best for comparing the effect of the different parameters.

3.2 Sobol method

The Sobol method is a quantitative method that gives the percentage of total output variance that each parameter accounts for. The method is a variance-based method to quantify the impact of uncertainties in random variables on the uncertainty of the model output. This method is more computationally expensive than the Morris method. The Sobol method for variance-based estimation is based on decomposition of the variance of a response to its variation sources. The Sobol method makes estimates of first-order sensitivity indices, higher–order indices and total indices. The first-order term represents the partial variance in the response due to the individual effects of a random variable. The higher-order terms show the interaction between two and more variables. The total effect relates to all direct and indirect variance from other variables.

3.3 Parameter variations for the model

The input parameters (Table 2) were selected as typical variation. The heat resistance for the roof varies from an old house with low level of thermal insulation to a highly insulated house. The length of the overhang varies from 10 to 90 cm as realistic values. The thermal resistance of the overhang is low as thermal insulation here is not common. The snow density varies from new snow as 80 kg/m^3 to older more compacted snow with 200 kg/m^3. The snow thickness ranges from 1 to 30 cm. The indoor temperature ranges from 5 °C in a room or attic with little heating to a heated room of 22 °C. The outdoor temperature ranges from -2 °C to -16 °C as typical in periods with snow on the roof and melting.

The analysis is based on at least 10,000 cases as the calculation is fast. Using other limits for the parameters will change the calculated indices, but in most cases this will not change the order for the important parameters.

Table 2. Uniform variation for the input parameters in the analysis

Parameter	Minimum	Maximum	Unit
Roof length	7	15	m
Roof heat resistance	1	5	m^{2}K/W
Overhang length	0.1	0.9	m
Overhang heat resistance	0.5	1	m^{2}K/W
Snow density	80	200	kg/m^3
Snow thickness	0.01	0.30	m
Temperature indoor	5	22	°C
Temperature outdoor	-16	-2	°C

3.4 Analysis of snow melting on roof

The first resulting parameter is the snow melting on the roof. Table 3 gives the Morris μ^* and σ values for melting. The μ^* values give a ranking for the influence of each parameter. Two parameters related to the overhang have a value of zero as they have no influence on the melting on the roof. In order the most important is the roof heat resistance, the Indoor temperature and finally the Snow thickness. The μ value for roof heat resistance and Snow density is negative. This indicates that an increase in these parameters will reduce the amount of melting water.

Table 3. Morris indices for snow melting

Parameter	μ^*	μ	σ
Roof length	0.432	0.432	0.781
Roof heat resistance	1.795	-1.795	2.526
Overhang length	0	0	0

Overhang heat resistance	0	0	0
Snow density	0.083	-0.083	0.119
Snow thickness	0.656	0.656	1.270
Temperature indoor	0.968	0.968	1.371
Temperature outdoor	0.361	0.361	0.518

Table 4 gives the Sobol indices for each parameter where S1 are the first order indices. The highest value is the roof heat resistance 0.56 and then the indoor temperature and finally the snow thickness. This is the same order as in the Morris analysis. The total Sobol indices ST gives the total effect of the interaction from two and more other variables. This will always be larger than the S1 indices. The most important is as before the roof heat resistance that increase from 0.55 to 0.76. The parameter with the lowest value is the Snow density, so an average value could have been used without much effect on the results. The Roof length and the Outdoor temperature have a low S1 value but if we include the higher order effects these clearly have more influence.

Table 4. Sobol indices for snow melting.

Parameter	S1	ST
Roof length	0.0190	0.0596
Roof heat resistance	0.5558	0.7624
Snow density	0.0021	0.0038
Snow thickness	0.0628	0.1479
Temperature indoor	0.1144	0.2568
Temperature outdoor	0.0247	0.0563

3.5 Analysis of water freezing on the overhang

The second resulting parameter is the water freezing on the overhang. Table 5 gives the Morris μ^* and σ values for freezing. The much lower values compared to melting is caused by the absolute value of the freezing being much lower than the melting. The most important factor is the outdoor temperature and after that, the snow thickness and finally the overhang length. But the next two parameters, i.e. the roof heat resistance and the indoor temperature, cannot be ignored. Here both the roof length and the snow density could be put in as average values.

Table 5. Morris indices for freezing

Parameter	μ^*	μ	σ
Roof length	0.003	0.003	0.015
Roof heat resistance	0.053	-0.053	0.102
Overhang length	0.060	0.060	0.080
Overhang heat resistance	0.019	-0.019	0.033

Snow density	0.012	-0.006	0.037
Snow thickness	0.064	0.045	0.101
Temperature indoor	0.034	0.034	0.081
Temperature outdoor	0.070	-0.031	0.105

Table 6 gives the Sobol indices for each parameter where the S1 is the first order indices. The highest value is the roof heat resistance 0.14 and second the overhang length and third the snow thickness. For the total Sobol indices ST most important is as before the roof heat resistance that increases from 0.14 to 0.48. In the case of freezing many parameters interact as for instance outdoor temperature that has a S1 of 0.03 and an ST 0.35. The total values of ST are very important for a realistic evaluation. The parameters with the lowest values are the snow density and the roof length, so an average value could have been used without much effect on the results.

Table 6. Sobol indices for freezing.

Parameter	S1	ST
Roof length	-0.0003	0.0053
Roof heat resistance	0.1414	0.4785
Overhang length	0.1259	0.2801
Overhang heat resistance	0.0198	0.0395
Snow density	0.0035	0.0048
Snow thickness	0.1238	0.4471
Temperature indoor	0.0742	0.3212
Temperature outdoor	0.0300	0.3539

3.6 Analysis of water dripping from the roof

The third resulting parameter analysis gives information on the water dripping from the roof. It is from this water that icicles can be formed – depending on the outdoor temperature, wind speed and the amount of water. Too much water could melt the icicles. Table 7 gives the Morris indices. The most important is the roof heat resistance and then the Indoor temperature and then the Snow thickness. The influence from the freezing on the overhang has nearly no effect, so the results are similar to those for melting.

Table 7. Morris indices for dripping/icicles

Parameter	μ^*	μ	σ
Roof length	0.514	0.514	0.918
Roof heat resistance	1.739	-1.739	2.332
Overhang length	0.061	-0.061	0.081
Overhang heat resistance	0.019	0.019	0.033

Snow density	0.078	-0.078	0.158
Snow thickness	0.731	0.731	1.412
Temperature indoor	1.058	1.058	1.574
Temperature outdoor	0.407	0.407	0.687

Table 8 gives the Sobol indices for each parameter where the S1 is the first order indices. The highest value is the roof heat resistance 0.55 and then the indoor temperature and then the snow thickness. This is the same order as found in the Morris analysis. The most important ST is as before the roof heat resistance that increase from 0.55 to 0.77. The roof length and the outdoor temperature have low S1 values but if we include the higher order effects these will have a more important influence as seen in ST. The parameter with the lowest value is the snow density, so an average value could have been used without much effect on the results. The influence of the overhang is very low and can be ignored.

Table 8. Sobol indices for dripping/icicles.

Parameter	S1	ST
Roof length	0.0198	0.0627
Roof heat resistance	0.5475	0.7728
Overhang length	0.0006	0.0018
Overhang heat resistance	0.0001	0.0003
Snow density	0.0014	0.0048
Snow thickness	0.0531	0.1429
Temperature indoor	0.1064	0.2605
Temperature outdoor	0.0307	0.0684

4 Example of use

The statistical analysis can be used for practical examples. In case of an old house, what is the best solution for reducing the risk of icicles? We cannot change the climate or the roof and overhang length. By using table 8 we can see that the largest total Sobol indices (ST) 0.77 is for the roof heat resistance. A better thermal insulation is the best solution. If this is not possible then reduce the indoor temperature, but the effect is much less as ST is 0.26. An alternative is to ventilate the attic with outdoor air if that is possible. In that case we use the attic temperature instead of the indoor temperature and the heat resistance R_r from the attic to the top of the roof. ms. A change in the overhang (which is typically difficult to do) as length and heat resistance has in practice no effect.

For a new house we could use a high thermal insulation in the roof to reduce the risk and if possible a longer overhang as melting water can freeze on the overhang, see section 2.1. But the most important is the thermal insulation and the indoor temperature.

More information on when icicles are growing and when icicles have the highest risk for falling down can be found in Nielsen [6] and [12].

5 Conclusions

The statistical analysis methods are a very useful method to find the most important factors in a model. For the melting of snow on roofs the most important factor is the heat resistance for the roof (ST = 0.76) and second most important factor is the indoor temperature (ST = 0.25) and third the snow thickness (ST = 0.15). For freezing is it interesting to note that overhang parameters are not the most important. For dripping is the important parameters the same as for melting.

To avoid dripping and icicles is it important to have a good thermal insulation and low indoor temperatures in an attic. In a house the indoor temperature will in most cases be decided by the occupants, so it is the thermal insulation that has to be good. As seen in section 3.1 the overhang is more important in a good insulated house.

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